

ECE 443/643 Test 1

October 10, 2011

1. Let the *raised-cosine pulse* $x(t)$ be defined by

$$x(t) = \begin{cases} \frac{1}{2}(1 + \cos(\frac{\pi t}{\tau})), & |t| < \frac{\tau}{2} \\ 0, & \text{else} \end{cases}$$

- (a) (5) Sketch $x(t)$.
(b) (15) Find $X(f) = \mathcal{F}\{x(t)\}$ (the continuous time Fourier transform of $x(t)$).
(c) (10) Find $\int_{-\infty}^{\infty} |X(f)|^2 df$.
2. (10) Let $T = 1\text{ms}$, $g(t) = \sum_{k=-\infty}^{\infty} G[k]e^{2\pi jkt/T}$,

$$G[k] = \begin{cases} \frac{1}{\pi jk}, & k \neq 0 \\ 0, & k = 0 \end{cases},$$

and

$$H(f) = \begin{cases} 2j, & 2.5 < f < 3.5 \text{ kHz} \\ -2j, & -3.5 < f < -2.5 \text{ kHz} \\ 0, & \text{else} \end{cases}.$$

Find the response $y(t)$ when $g(t)$ is input into the system with transfer function $H(f)$.

3. (10) The signal $x(t) = 2\text{sinc}(40t)$ is input into an LTI system with transfer function $H(f)$. If the output is $y(t) = 20\text{sinc}(40t - 200)$. Find $H(f)$ for $|f| \leq 30$.
4. (15) Find the Fourier series coefficients of sawtooth wave

$$x(t) = \begin{cases} \frac{1}{2} - t, & 0 < t < 1 \\ x(t - n), & \text{else, } n \in \mathbb{Z} \end{cases}$$

5. (10) Let $x(t) = g(-3(t - 2)) + g^\dagger(t) - \Re(g(t)e^{j4\pi t})$. Find $X(f)$ in terms of $G(f)$.